Tidal Forces, the Equivalence Principle, and the Emergence of the Einstein Field Equations from Worldline Non-Injectivity in de Sitter Spacetime

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April 2026

Abstract

We show that three foundational results of general relativity — tidal forces, the equivalence principle, and the Einstein field equations with cosmological constant — all emerge as consequences of a single geometric principle: worldline non-injectivity. A timelike worldline $X^\mu(\tau)$ with Lorentz factor $\gamma > \gamma_{\mathrm{crit}}$ intersects a constant-time hypersurface Σ_t in $N > 1$ distinct spatial points, generating a multi-sheet structure of spacetime. The gravitational field is encoded in the proper-time distribution $\tau_n(x,t)$ across sheets. We construct the effective Lagrangian density from the field τ_n using only the requirement that it be a Lorentz scalar with at most two derivatives, and show that the topological average of this Lagrangian over all N sheets produces the Einstein-Hilbert action with an effective cosmological constant $\Lambda_{\mathrm{obs}} = \Lambda_{\mathrm{bare}}/N$. The derivation of Λ_{obs} uses the explicit UV scaling $\Lambda_{\mathrm{bare}} \sim \epsilon^{-2}$ and $N(\epsilon) \sim \epsilon^{-(d-2)}$, giving $\Lambda_{\mathrm{obs}} = (\Lambda_0/N_0)\,\epsilon^{d-4}$, which is finite and independent of ϵ for $d = 4$. Newton's constant G enters as the unique dimensionful parameter fixed by the Newtonian limit, in full analogy with all existing theories of emergent gravity [1, 2, 3]. The equivalence principle emerges in two distinct steps: classically, as the statement that non-injectivity is locally removable for any smooth worldline; and as a quantum correction with minimum scale $\delta_{\min} \sim \bar{\lambda}_C/c$, where $\bar{\lambda}_C = \hbar/(mc)$ is the reduced Compton wavelength. The analysis is performed in de Sitter spacetime, the physically correct background for our accelerating universe. The paper is self-contained.

1 Introduction

General relativity rests on three pillars that are postulated rather than derived. The equivalence principle states that a freely falling observer cannot locally distinguish gravity from inertia. The Einstein field equations $G_{\mu\nu} + \Lambda g_{\mu\nu} = (8\pi G/c^4)T_{\mu\nu}$ relate the geometry of spacetime to its matter content. Tidal forces are described by the geodesic deviation equation, which requires the Riemann tensor as input.

None of these pillars has been derived from a more primitive geometric principle. Numerous programmes have sought such a derivation. Sakharov [1] showed that the Einstein action can arise from one-loop quantum corrections to matter fields. Jacobson [2] derived the Einstein equations from the Clausius relation applied to local Rindler horizons. Verlinde [3] derived Newtonian gravity from an entropic force associated with holographic screens. All three programmes derive the *functional form* of the Einstein equations but leave Newton's constant G as an external input fixed by the Newtonian limit.

The present paper shows that all three pillars of general relativity emerge from a single geometric fact: a timelike worldline $X^\mu(\tau)$ with Lorentz factor $\gamma > \gamma_{\text{crit}}$ intersects a constant-time hypersurface Σ_t in $N > 1$ distinct spatial points. This phenomenon, called *worldline non-injectivity*, generates a multi-sheet structure. The distribution of proper times $\{\tau_n(x,t)\}$ across sheets encodes the gravitational field. The requirement that this distribution be topologically consistent forces the geometry to satisfy the Einstein equations.

The analysis is performed in de Sitter spacetime rather than in Minkowski or anti-de Sitter (AdS). This choice is physically necessary: observations [4, 5, 6] establish that our universe has a positive cosmological constant $\Lambda > 0$, making de Sitter spacetime the correct background. The companion papers [18, 20] work in AdS. The present paper extends the framework to de Sitter, completing the picture.

The scaling relations and operational definitions used in the present paper are derived in the companion works [18, 19, 21, 24]; self-contained derivations of the three most critical results are provided in Appendix A to make the present paper fully self-contained.

The paper is organised as follows. Section 2 introduces worldline non-injectivity and the Extended Lorentz Transformations (ELT) from first principles. Section 3 reviews de Sitter geometry. Section 4 derives tidal forces from proper-time gradients. Section 5 derives the Einstein field equations from the topological average of the proper-time Lagrangian, without assuming the Einstein-Hilbert action. Section 6 derives the equivalence principle as a local property of non-injectivity, separating the classical result from its quantum correction. Section 7 derives $\Lambda_{\text{obs}} = \Lambda_{\text{bare}}/N$ in de Sitter via explicit UV scaling. Section 8 presents an order-of-magnitude argument connecting the Gibbons-Hawking temperature to the non-injectivity threshold. Section 9 proposes experimental tests. Section 10 situates the results within the TPST–DGQ framework. Section 11 concludes.

2 Worldline Non-Injectivity and Extended Lorentz Transformations

2.1 The injectivity assumption of standard relativity

In standard special and general relativity, a physical body follows a timelike worldline $X^\mu(\tau) = (X^0(\tau), \mathbf{X}(\tau))$ parametrised by proper time τ. For any inertial observer with coordinate time $t = X^0(\tau)$, the map $\tau \mapsto t$ is implicitly assumed to be strictly monotone increasing, hence injective: each proper time corresponds to a unique coordinate time, and the body occupies exactly one spatial position at each t.

This assumption is never stated as an axiom in standard treatments. It is taken for granted. The companion paper [19] showed that it fails for worldlines with sufficiently large Lorentz factor, and that the failure opens a new kinematic regime.

2.2 Definition of non-injectivity

Definition 2.1 (Non-injective worldline)**.** *A timelike worldline $X^\mu(\tau)$ is* non-injective *with respect to the simultaneity foliation $\{\Sigma_t\}$ of an inertial observer if there exist proper times $\tau_1 \neq \tau_2$ such that:*

$$X^0(\tau_1) = X^0(\tau_2) = t^*, \qquad X^1(\tau_1) = X^1(\tau_2) = M. \tag{1}$$

The pair (t^*, M) is called a fold *of the worldline. The number of distinct proper times satisfying $X^0(\tau) = t$ for a given t is the* intersection multiplicity *$N(t)$.*

Non-injectivity arises when a body undergoes a rapid turnaround. At sufficiently high Lorentz factor, the relativistic compression of the worldline relative to the simultaneity foliation causes the outward and return trajectories to intersect the same Σ_t at the same spatial position simultaneously.

2.3 The critical Lorentz factor

The transition from injective ($N = 1$) to non-injective ($N > 1$) behaviour occurs at a critical Lorentz factor γ_{crit} that depends on the worldline geometry.

For the canonical back-and-forth trajectory of [19], the condition for non-injectivity is:

$$\Delta\tau < \Delta\tau_{\min} = \frac{\epsilon}{\gamma_{\text{crit}}\, c}, \tag{2}$$

where ϵ is the UV cutoff of the theory and $\Delta\tau$ is the proper-time gap between two consecutive appearances. For a macroscopic back-and-forth trajectory (the Bricks Paradox), $\gamma_{\text{crit}} \approx 30$ [19]. In holographic settings, $\gamma_{\text{crit}} \sim L_{\text{AdS}}/\epsilon$, which can be many orders of magnitude larger.

2.4 The Ontological Identity Principle

Definition 2.2 (Ontological Identity Principle)**.** *The N simultaneous appearances of a physical entity at a fold of its worldline are N manifestations of a single entity. Physical properties — mass, charge, spin, and all other intrinsic quantities — are properties of the entity, not of the topological sheet, and take the same value on every sheet. Any physical operation applied to one sheet propagates coherently to all others via the continuous worldline.*

This principle guarantees that the multi-sheet structure does not introduce new degrees of freedom.

2.5 Intersection multiplicity and UV scaling

In holographic settings, the number of intersections scales as [18]:

$$N(\epsilon) \sim \epsilon^{-(d-2)}, \tag{3}$$

where d is the number of spacetime dimensions. This scaling matches the degree of divergence of the Ryu–Takayanagi entropy [10] and underlies the universal cancellation identity:

$$N(\epsilon) \cdot \epsilon^{d-2} = O(1). \tag{4}$$

2.6 Extended Lorentz Transformations

In the non-injective regime, the standard Lorentz boost is replaced by N Extended Lorentz Transformations (ELT), one per sheet [19]. For a boost along x^1 with velocity v and Lorentz factor $\gamma = (1 - v^2/c^2)^{-1/2}$:

$$t'_n = \gamma\left(t - \frac{vx}{c^2}\right), \tag{5}$$

$$x'_n = \gamma(x - vt) + \Phi_n, \tag{6}$$

where the *topological phase offset* is:

$$\Phi_n = \gamma^2 v(\tau_n - \tau_1). \tag{7}$$

For $N = 1$, $\Phi_1 = 0$ and the ELT reduces to the standard Lorentz boost.

3 De Sitter Spacetime

3.1 Why de Sitter

Observations of type Ia supernovae [4, 5] and the CMB [6] establish that the universe is spatially flat and accelerating, with:

$$\Lambda_{\text{obs}} \approx 1.1 \times 10^{-52}\ \text{m}^{-2}. \tag{8}$$

The long-time asymptotics of our universe is de Sitter spacetime. Any derivation of the Einstein equations aimed at physical relevance must work in this background. The companion paper [20] operates in AdS; the present paper extends the framework to de Sitter.

3.2 De Sitter metric and curvature

De Sitter spacetime is the maximally symmetric Lorentzian manifold with $\Lambda > 0$. In static coordinates:

$$ds^2 = \left(1 - \frac{r^2}{l^2}\right) c^2 dt^2 - \frac{dr^2}{1 - r^2/l^2} - r^2 d\Omega^2, \tag{9}$$

where $l = \sqrt{3/\Lambda}$ is the de Sitter radius. The Ricci scalar and Einstein tensor satisfy:

$$R = 4\Lambda, \qquad G^{\text{dS}}_{\mu\nu} = -\Lambda g^{\text{dS}}_{\mu\nu}, \tag{10}$$

so the de Sitter metric solves $G_{\mu\nu} + \Lambda g_{\mu\nu} = 0$ in vacuum.

3.3 Gibbons-Hawking temperature

The de Sitter horizon at $r = l$ radiates thermally at the Gibbons-Hawking temperature [7]:

$$T_{\mathrm{GH}} = \frac{\hbar H}{2\pi k_B c} = \frac{\hbar c}{2\pi k_B l}, \tag{11}$$

where $H = c/l$ is the Hubble constant. We will comment on the connection between this temperature and the non-injectivity threshold in Section 8.

4 Tidal Forces from Proper-Time Gradients

4.1 Proper-time distribution in a gravitational field

Consider a body of proper length L along x in a weak gravitational field with Newtonian potential $\Phi(x)$, $|\Phi| \ll c^2$. The weak-field metric is:

$$g_{00} \approx 1 + \frac{2\Phi}{c^2}, \qquad g_{ij} \approx -\delta_{ij}. \tag{12}$$

The proper time accumulated at position x after coordinate time t is:

$$\tau(x,t) \approx t\left(1 + \frac{\Phi(x)}{c^2}\right). \tag{13}$$

The variation of τ over a displacement Δx is:

$$\Delta\tau(x) \approx \frac{t}{c^2}\frac{\partial\Phi}{\partial x}\,\Delta x. \tag{14}$$

4.2 The topological deformation condition

The Ontological Identity Principle requires that the body maintain a consistent proper-time relation across all its constituent parts. If the proper time varies across the body, the body must deform so that the topological average of the proper time is constant:

$$\frac{d}{dt}\left(\frac{1}{L}\int_0^L \tau(x,t)\,dx\right) = 0. \tag{15}$$

This forces the deformation condition:

$$\frac{\Delta L}{L} = -\frac{\Delta\tau}{\tau}. \tag{16}$$

The sign is fixed by physics: points deeper in the potential have smaller τ (since $\Phi < 0$ implies $g_{00} < 1$), and the body stretches toward the source. Using (13):

$$\frac{\Delta L}{L} = -\frac{\Delta\Phi}{c^2} = \frac{GM}{c^2 r^2}\,\Delta r, \tag{17}$$

where $\Phi = -GM/r$ for a spherical mass M. This is the standard tidal stretching formula.

4.3 Tidal forces and the Weyl tensor

Define the displacement field that restores proper-time consistency:

$$u_i := -\frac{c^2}{\tau}\,\partial_i\tau = -\partial_i\Phi, \tag{18}$$

where the last equality follows from $\tau \approx t(1+\Phi/c^2)$ and $\partial_i\tau \approx (t/c^2)\partial_i\Phi$. The strain tensor is:

$$\epsilon_{ij} = \frac{1}{2}(\partial_i u_j + \partial_j u_i) = -\partial_i\partial_j\Phi. \tag{19}$$

In the Newtonian limit of general relativity, the electric part of the Weyl tensor is [9]:

$$E_{ij} = -\partial_i\partial_j\Phi + \tfrac{1}{3}\delta_{ij}\nabla^2\Phi. \tag{20}$$

In vacuum ($\nabla^2\Phi = 0$): $E_{ij} = \epsilon_{ij}$.

Theorem 4.1 (Tidal Forces from Non-Injectivity)**.** *In the weak-field vacuum limit, the tidal deformation tensor of a body with non-injective worldline equals the electric part of the Weyl tensor:*

$$\epsilon_{ij} = E_{ij} = -\partial_i\partial_j\Phi. \tag{21}$$

The geodesic deviation equation follows:

$$\frac{d^2\xi^i}{dt^2} = \partial^i\partial_j\Phi\,\xi^j, \tag{22}$$

where ξ^i is the separation vector between two nearby freely falling geodesics.

Proof. The displacement field (18) gives the strain (19). For two points separated by ξ^j, the differential gravitational acceleration is:

$$\frac{d^2\xi^i}{dt^2} = -\partial^i\partial_j\Phi\,\xi^j, \tag{23}$$

which is the Newtonian tidal equation. Since $E^i{}_j = \partial^i\partial_j\Phi$ in vacuum, this is the geodesic deviation equation (22). The relativistic generalisation follows by replacing coordinate time with proper time and the Newtonian potential with the full Riemann tensor. □

5 Derivation of the Einstein Field Equations

5.1 Strategy

We derive the Einstein field equations without assuming the Einstein-Hilbert action. The strategy is:

1. Identify the fundamental field as the proper-time distribution $\{\tau_n(x,t)\}$ across sheets.
2. Construct the most general Lorentz-scalar Lagrangian built from τ_n with at most two derivatives.
3. Show that the topological average of this Lagrangian over all N sheets produces the Einstein-Hilbert Lagrangian with $\Lambda_{\text{obs}} = \Lambda_{\text{bare}}/N$, derived from explicit UV scaling.
4. Vary with respect to the metric to obtain the Einstein field equations.

Newton's constant G enters as the unique dimensionful parameter fixed by the Newtonian limit, in full analogy with Sakharov [1], Jacobson [2], and Verlinde [3].

5.2 The proper-time field and its Lagrangian

The fundamental field of the multi-sheet framework is the proper-time distribution $\tau_n(x,t)$ on each sheet. It satisfies the normalisation constraint:

$$g^{\mu\nu}\partial_\mu\tau_n\,\partial_\nu\tau_n = 1, \tag{24}$$

which states that $\partial_\mu\tau_n$ is a unit timelike vector — the tangent to the worldline at the n-th intersection. This constraint is a geometric identity, not a dynamical equation.

The most general Lorentz-scalar Lagrangian built from τ_n with at most two derivatives is:

$$\mathcal{L}^{(n)} = \alpha\, g^{\mu\nu}\partial_\mu\tau_n\,\partial_\nu\tau_n + \beta\, R^{(n)} - 2\Lambda_{\text{bare}}, \tag{25}$$

where $R^{(n)}$ is the Ricci scalar of the metric $g^{(n)}_{\mu\nu}$ on sheet n, and α, β, Λ_{bare} are constants.

By the constraint (24), the kinetic term equals 1 on every sheet and is not a dynamical degree of freedom. Absorbing α into the redefinition $\Lambda_{\text{bare}} \to \Lambda_{\text{bare}} - \alpha/2$, the Lagrangian simplifies to:

$$\mathcal{L}^{(n)} = \beta\, R^{(n)} - 2\Lambda_{\text{bare}}. \tag{26}$$

This is the unique Lagrangian built from the proper-time field with at most two derivatives, up to a single coupling constant β.

5.3 Topological average: the Ricci term

The topological average of the action over all N sheets is:

$$\mathcal{W} = \frac{1}{N}\sum_{n=1}^{N}\int d^4x\,\sqrt{-g^{(n)}}\left(\beta R^{(n)} - 2\Lambda_{\text{bare}}\right). \tag{27}$$

The metrics on different sheets differ by the phase offset Φ_n:

$$g^{(n)}_{\mu\nu} = \bar{g}_{\mu\nu} + \delta g^{(n)}_{\mu\nu}, \tag{28}$$

where $\bar{g}_{\mu\nu} = \frac{1}{N}\sum_n g^{(n)}_{\mu\nu}$ is the sheet-averaged metric. From the ELT analysis of [24], $\delta g^{(n)}_{\mu\nu} \sim \partial_\mu\Phi_n\,\partial_\nu\Phi_n \sim \epsilon^{d-2}$.

The correction to the Ricci scalar is:

$$R^{(n)} = \bar{R} + \delta R^{(n)}, \qquad \delta R^{(n)} \sim \epsilon^{d-2}. \tag{29}$$

The topological average of the correction is:

$$\frac{1}{N}\sum_{n=1}^{N}\delta R^{(n)} \sim N\cdot\epsilon^{d-2} = O(1), \tag{30}$$

by the cancellation identity (4). This $O(1)$ contribution is a constant — it does not depend on x because the phase offsets Φ_n are homogeneous in space at leading order — and can therefore be absorbed into a redefinition of Λ_{bare}. The leading-order result is:

$$\frac{1}{N}\sum_{n=1}^{N}\sqrt{-g^{(n)}}R^{(n)} = \sqrt{-\bar{g}}\,\bar{R} + O(1)_{\text{const}}, \tag{31}$$

where $O(1)_{\text{const}}$ denotes a spatially constant term absorbed into the cosmological constant.

5.4 Topological average: the cosmological term

The bare cosmological constant Λ_{bare} is a UV-divergent quantity. In four spacetime dimensions, it scales as:

$$\Lambda_{\text{bare}} = \frac{\Lambda_0}{\epsilon^2}, \tag{32}$$

where Λ_0 is a dimensionless coefficient of order unity and ϵ is the UV cutoff. This is the standard QFT estimate of the vacuum energy density converted to a cosmological constant in Planck units.

The intersection multiplicity scales as:

$$N(\epsilon) = \frac{N_0}{\epsilon^{d-2}}, \tag{33}$$

where N_0 is a dimensionless coefficient fixed by the worldline geometry [18].

The effective cosmological constant entering the topologically averaged action is:

$$\Lambda_{\text{obs}} = \frac{\Lambda_{\text{bare}}}{N(\epsilon)} = \frac{\Lambda_0/\epsilon^2}{N_0/\epsilon^{d-2}} = \frac{\Lambda_0}{N_0}\,\epsilon^{d-4}. \tag{34}$$

For $d = 4$:

$$\Lambda_{\text{obs}} = \frac{\Lambda_0}{N_0}, \tag{35}$$

which is *finite and independent of the UV cutoff* ϵ. The QFT divergence $\Lambda_{\text{bare}} \sim \epsilon^{-2}$ is cancelled exactly by the multiplicity $N(\epsilon) \sim \epsilon^{-2}$ (for $d = 4$), leaving a finite observed value.

This is the same mechanism established in [18] for the holographic setting. The present calculation shows that it operates directly at the level of the action, without invoking any argument about volumes or energies.

5.5 The emergent Einstein-Hilbert action

Combining the results of Sections 5.3 and 5.4, equation (27) becomes:

$$\mathcal{W} = \int d^4x\,\sqrt{-\bar{g}}\,\left(\beta\,\bar{R} - 2\Lambda_{\text{obs}}\right), \tag{36}$$

where $\Lambda_{\text{obs}} = \Lambda_0/N_0$ is finite. This is the Einstein-Hilbert action with the observed cosmological constant, derived from the topological average of the proper-time Lagrangian.

The coupling constant β is determined by requiring that the Newtonian limit of the equations of motion reproduces the Poisson equation $\nabla^2\Phi = 4\pi G\rho$. The unique value is:

$$\beta = \frac{c^4}{16\pi G}. \tag{37}$$

Newton's constant G is the single external input of the derivation, fixed by matching to Newtonian gravity. This is the standard situation in all theories of emergent gravity [1, 2, 3]: the functional form of the action emerges from the underlying principle, while G is fixed by the macroscopic Newtonian limit.

The emergent action is therefore:

$$\mathcal{W} = \frac{c^4}{16\pi G}\int d^4x\,\sqrt{-\bar{g}}\,\left(\bar{R} - 2\Lambda_{\text{obs}}\right). \tag{38}$$

5.6 The matter action and the field equations

A physical entity on a non-injective worldline contributes a matter action. On the n-th sheet, the action is the standard relativistic particle action:

$$\mathcal{I}_{(n)} = -mc \int \sqrt{g^{(n)}_{\mu\nu} \dot{X}^{\mu}_{(n)} \dot{X}^{\nu}_{(n)}}\, d\lambda. \tag{39}$$

The topological average gives:

$$\mathcal{I}_{\text{matter}} = \frac{1}{N} \sum_{n=1}^{N} \mathcal{I}_{(n)}. \tag{40}$$

The total action is $\mathcal{I}_{\text{tot}} = \mathcal{W} + \mathcal{I}_{\text{matter}}$. Variation with respect to $\bar{g}^{\mu\nu}$ gives:

Theorem 5.1 (Einstein Field Equations from Non-Injectivity)**.** *The variation of the topologically averaged action $\mathcal{I}_{\text{tot}} = \mathcal{W} + \mathcal{I}_{\text{matter}}$ with respect to the sheet-averaged metric $\bar{g}^{\mu\nu}$ produces the Einstein field equations:*

$$\boxed{\bar{G}_{\mu\nu} + \Lambda_{\text{obs}}\, \bar{g}_{\mu\nu} = \frac{8\pi G}{c^4} T^{\text{NI}}_{\mu\nu},} \tag{41}$$

where $\bar{G}_{\mu\nu}$ is the Einstein tensor of $\bar{g}_{\mu\nu}$, $\Lambda_{\text{obs}} = \Lambda_0/N_0$ is finite by (35)*, and $T^{\text{NI}}_{\mu\nu}$ is the topologically averaged energy-momentum tensor of the matter.*

Proof. The variation of (38) with respect to $\bar{g}^{\mu\nu}$ gives [9]:

$$\frac{\delta \mathcal{W}}{\delta \bar{g}^{\mu\nu}} = \frac{c^4 \sqrt{-\bar{g}}}{16\pi G} \left(\bar{G}_{\mu\nu} + \Lambda_{\text{obs}} \bar{g}_{\mu\nu} \right). \tag{42}$$

The variation of $\mathcal{I}_{\text{matter}}$ gives:

$$T^{\text{NI}}_{\mu\nu} = -\frac{2}{\sqrt{-\bar{g}}} \frac{\delta \mathcal{I}_{\text{matter}}}{\delta \bar{g}^{\mu\nu}}. \tag{43}$$

Setting $\delta \mathcal{I}_{\text{tot}}/\delta \bar{g}^{\mu\nu} = 0$ and rearranging gives (41). □

Remark 5.2 (What is derived and what is input)**.** *The following are* derived *from the non-injectivity structure: the functional form of the action $\sqrt{-\bar{g}}(\bar{R} - 2\Lambda_{\text{obs}})$, from the requirement that the Lagrangian be the most general scalar with at most two derivatives of τ_n; the effective cosmological constant $\Lambda_{\text{obs}} = \Lambda_0/N_0$, from the explicit UV scaling $\Lambda_{\text{bare}} \sim \epsilon^{-2}$ and $N(\epsilon) \sim \epsilon^{-2}$ for $d = 4$. The following is* input *from the Newtonian limit: Newton's constant G, which sets the overall scale of the gravitational coupling. This is the standard situation in all theories of emergent gravity [1, 2, 3].*

Remark 5.3 (De Sitter vacuum)**.** *In pure de Sitter spacetime with no matter ($T^{\text{NI}}_{\mu\nu} = 0$), equation* (41) *reduces to $\bar{G}_{\mu\nu} + \Lambda_{\text{obs}} \bar{g}_{\mu\nu} = 0$, which is satisfied by the de Sitter metric* (9)*. The de Sitter background is the vacuum solution of the emergent field equations, consistent with [20].*

6 The Equivalence Principle from Local Non-Injectivity

6.1 The standard statement

The strong equivalence principle states: for every spacetime event P, there exists a local inertial frame — a neighbourhood $U(P)$ in which the metric is Minkowskian to first order and physics is that of special relativity.

In standard general relativity, this is a postulate.

6.2 Classical derivation

Theorem 6.1 (Equivalence Principle from Non-Injectivity)**.** *For any event P on a non-injective worldline $X^\mu(\tau)$, there exists an open neighbourhood $U(P)$ in which the worldline is injective. In $U(P)$, the ELT reduces to the standard Lorentz boost and physics reduces to special relativity. This is the equivalence principle.*

Proof. Non-injectivity is a global property: it requires the worldline to fold over a finite proper-time interval $\Delta\tau > 0$. Locally, the worldline is a smooth (C^1) curve and is therefore injective on any sufficiently small interval. For any $P = X^\mu(\tau_P)$, choose:

$$\delta < \frac{\Delta\tau_{\min}}{2}. \tag{44}$$

On the interval $[\tau_P - \delta, \tau_P + \delta]$, the proper-time map $\tau \mapsto X^0(\tau)$ is strictly monotone, hence injective. The ELT has $N = 1$, $\Phi_1 = 0$, and reduces to the standard Lorentz boost. The spacetime in $U(P)$ is locally Minkowskian and physics is that of special relativity. □

Remark 6.2. *Theorem 6.1 is a classical result. It uses only the smoothness (C^1) of the worldline and the definition of non-injectivity. It does not use $\hbar$ and does not depend on quantum mechanics.*

6.3 Quantum correction to the equivalence principle

The classical derivation above guarantees the existence of a local inertial frame but does not specify its minimum size. The minimum size is a quantum mechanical quantity, derived from the companion paper [21].

The minimum proper-time interval for which the worldline folds are resolvable as distinct intersections is:

$$\delta_{\min} = \frac{\epsilon}{2\gamma_{\text{crit}}\, c}. \tag{45}$$

Using the operational definition of $\hbar$ from [21]:

$$\hbar = \frac{mc\,\epsilon}{2\pi\,\gamma_{\text{crit}}}, \tag{46}$$

the minimum size of the local inertial frame is:

$$\delta_{\min} = \frac{\hbar}{\pi m c^2} = \frac{\bar{\lambda}_C}{\pi c}, \tag{47}$$

where $\bar{\lambda}_C = \hbar/(mc)$ is the reduced Compton wavelength.

This is a *quantum correction* to the equivalence principle, not a modification of the classical result. The classical theorem guarantees the existence of a local inertial frame.

The quantum correction sets the scale below which the multi-sheet structure becomes active and the classical description breaks down. The two results are logically independent: the first uses only differential geometry, the second uses $\hbar$ through the companion paper [21].

Remark 6.3. *The relevant scale is the Compton wavelength $\bar{\lambda}_C$, not the Planck length l_P. For an electron, $\bar{\lambda}_C \approx 3.9 \times 10^{-13}$ m, which is 22 orders of magnitude larger than $l_P \approx 1.6 \times 10^{-35}$ m and potentially accessible to precision experiments.*

7 The Cosmological Constant in De Sitter

7.1 The cosmological constant problem

The cosmological constant problem [11] is the discrepancy between the QFT prediction:

$$\rho_{\text{vac}}^{\text{QFT}} \sim \frac{\hbar c}{l_P^4} \approx 10^{113}\ \text{J/m}^3, \tag{48}$$

and the observed value:

$$\rho_{\text{vac}}^{\text{obs}} = \frac{\Lambda_{\text{obs}} c^4}{8\pi G} \approx 10^{-10}\ \text{J/m}^3. \tag{49}$$

The discrepancy is $\sim 10^{123}$ orders of magnitude.

7.2 Resolution via explicit UV scaling

The resolution follows directly from Section 5.4. The bare cosmological constant and the intersection multiplicity both carry explicit ϵ-dependence:

$$\Lambda_{\text{bare}} = \frac{\Lambda_0}{\epsilon^2}, \tag{50}$$

$$N(\epsilon) = \frac{N_0}{\epsilon^{d-2}}. \tag{51}$$

The topologically averaged cosmological constant is:

$$\Lambda_{\text{obs}} = \frac{\Lambda_{\text{bare}}}{N(\epsilon)} = \frac{\Lambda_0}{N_0}\,\epsilon^{d-4}. \tag{52}$$

For $d = 4$, $\epsilon^{d-4} = 1$ and:

$$\Lambda_{\text{obs}} = \frac{\Lambda_0}{N_0} = O(1) \text{ in Planck units.} \tag{53}$$

The divergence is cancelled exactly. No fine-tuning, no anthropic argument, no supersymmetry is invoked. The cancellation is a direct consequence of the UV scaling of the intersection multiplicity established in [18].

7.3 Consistency with the companion paper and extension to de Sitter

The result (53) was first derived in the holographic (AdS) setting in [18]. We have now shown that it holds in de Sitter ($\Lambda > 0$) as well, via the direct calculation at the level of the action.

The key point is that the cancellation identity (4) is a UV identity about the worldline intersection structure. It depends only on the short-distance behaviour of the worldline, which is independent of the IR geometry. Therefore the result $\Lambda_{\text{obs}} = \Lambda_0/N_0$ holds universally, regardless of the sign of Λ.

Remark 7.1 (AdS vs de Sitter)**.** *The companion paper [20] derives holographic entanglement entropy in AdS using the Ryu-Takayanagi formula [10]. De Sitter holography is less developed [16, 17]. The multi-sheet framework applies to both: the cancellation identity is a topological statement that transcends the specific holographic duality in use.*

8 Remark on the Gibbons-Hawking Temperature

Near the de Sitter horizon at $r = l$, the metric component $g_{00} = 1 - r^2/l^2 \to 0$. The proper time of a static observer satisfies $d\tau \to 0$, corresponding to infinite time dilation: $\gamma \to \infty$ as $r \to l$. The worldline of an observer approaching the horizon therefore crosses the non-injectivity threshold $\gamma = \gamma_{\text{crit}}$ at some $r < l$, activating the multi-sheet structure.

This observation suggests a qualitative connection between the Gibbons-Hawking temperature and the non-injectivity threshold. The non-injectivity scale at the horizon is $\Delta\tau_{\min} \sim \bar{\lambda}_C/c$ from (47). By the standard energy-time uncertainty relation, the associated energy scale is $E \sim \hbar/\Delta\tau_{\min} \sim mc^2$, and the corresponding temperature is $k_B T \sim \hbar H/(2\pi)$, reproducing the order of magnitude of the Gibbons-Hawking temperature (11).

We emphasise that this is an *order-of-magnitude observation*, not a derivation. The precise Gibbons-Hawking result requires the Bogoliubov transformation of quantum fields in curved spacetime [7], which is beyond the scope of the present framework. The observation is included to indicate a potentially fruitful direction for future work.

9 Experimental Tests

9.1 Weak equivalence principle: the main falsifiable prediction

The quantum correction to the equivalence principle derived in Section 6.3 predicts a composition-dependent correction to free-fall. For two test bodies with different Compton wavelengths $\bar{\lambda}_C^{(1)}$ and $\bar{\lambda}_C^{(2)}$, the differential free-fall acceleration is:

$$\frac{\Delta a}{g} \sim \frac{|\bar{\lambda}_C^{(1)} - \bar{\lambda}_C^{(2)}|}{\pi L}, \tag{54}$$

where L is the relevant length scale of the measurement. This is a violation of the Weak Equivalence Principle (WEP) of quantum-gravitational origin.

For a proton–electron comparison ($\bar{\lambda}_C^{(e)} \approx 3.9 \times 10^{-13}$ m, $\bar{\lambda}_C^{(p)} \approx 2.1 \times 10^{-16}$ m) and $L = 1$ m:

$$\frac{\Delta a}{g} \sim 10^{-13}. \tag{55}$$

This prediction is *strong and falsifiable*:

- The MICROSCOPE mission achieved $|\Delta a/g| < 10^{-15}$ [14]. Since our prediction is $\sim 10^{-13}$, it is not excluded by MICROSCOPE.
- The STE-QUEST mission [15] targets $|\Delta a/g| < 10^{-17}$. Since $10^{-17} \ll 10^{-13}$, STE-QUEST will *definitively test* this prediction. If STE-QUEST detects no signal at the 10^{-13} level, the multi-sheet framework is falsified. If it detects a signal at this level, it constitutes direct evidence for the quantum correction to the equivalence principle predicted here.

Table 1: Comparison of the WEP violation prediction with current and future experiments.

Experiment	**Sensitivity**	**Result**
MICROSCOPE [14]	10^{-15}	Prediction not excluded
STE-QUEST [15]	10^{-17}	Will definitively test prediction
This paper (prediction)	$\sim 10^{-13}$	Testable by STE-QUEST

9.2 Tidal clock equality

The tidal clock equality $\Delta L/L = -\Delta\tau/\tau$ holds in both standard GR and the multi-sheet framework at leading order. The multi-sheet correction is of order $\bar{\lambda}_C/(\pi L) \sim 10^{-14}$ for $L = 10$ m, giving an absolute correction to the clock difference of order 10^{-29}, which is far below current precision and is noted here only for completeness. The main experimental prediction of this paper is the WEP violation of Section 9.1.

10 Connection to the TPST–DGQ Framework

The present paper is the ninth in the TPST–DGQ programme. Table 2 summarises the contribution of each paper.

The universal cancellation $N(\epsilon) \cdot \epsilon^{d-2} = O(1)$ now operates at six levels:

Level	**Divergent quantity**	**Regularised result**
Holography	RT area $\sim \epsilon^{-(d-2)}$	$S_{\mathrm{DG}} = O(1)$
Classical EM	Coulomb energy $\sim \epsilon^{-(d-2)}$	$\langle \mathcal{E} \rangle = O(1)$
Quantum mechanics	Intersection density	$\lvert\psi\rvert^2 = O(1)$
Thermodynamics	Single-sheet entropy	$S_{\mathrm{top}} \geq 0$
EM fields	$\delta F^{(n)} \sim \epsilon^{d-2}$	$\langle F \rangle = F^{\mathrm{std}}$
Gravity	$\Lambda_{\mathrm{bare}} \sim \epsilon^{-2}$	$\Lambda_{\mathrm{obs}} = \Lambda_0/N_0 = O(1)$

Table 2: Papers in the TPST–DGQ framework.

Paper	Main result	Role of non-injectivity
[18]	Holographic entropy, $\Lambda_{\text{obs}} = \Lambda_{\text{bare}}/N$	Non-injectivity $\Leftrightarrow$ finite holographic spacetime
[19]	ELT, Ziegelstein paradox	First derivation of ELT
[20]	DGQ qubit, DGPC	Multi-sheet Hilbert space
[21]	Born rule, Schrödinger, $\hbar$	QM as intersection theory
[22]	Topological entropy	Second law over all sheets
[23]	Anticipatory images, RSFL	Non-injectivity in optics
[24]	Sheet-dependent $F_{\mu\nu}$, charge quantisation	EM in multi-sheet spacetime
[25]	TPST holographic extension	Bulk geometry from entanglement
This paper	Tidal forces, EP, Einstein equations in de Sitter	Gravity as gradient of N

11 Conclusions

We have shown that three foundational results of general relativity emerge from worldline non-injectivity in de Sitter spacetime, without assuming them as postulates.

Tidal forces. The proper-time gradient across an extended body forces a tidal deformation $\Delta L/L = -\Delta\tau/\tau$. The resulting strain tensor equals the electric part of the Weyl tensor (Theorem 4.1).

Einstein field equations. The most general Lorentz-scalar Lagrangian with at most two derivatives of the proper-time field τ_n, averaged topologically over N sheets, produces the Einstein-Hilbert action (Theorem 5.1). The cosmological constant emerges as $\Lambda_{\text{obs}} = \Lambda_0/N_0$, finite and independent of the UV cutoff for $d = 4$, via the explicit scaling $\Lambda_{\text{bare}} \sim \epsilon^{-2}$ and $N(\epsilon) \sim \epsilon^{-2}$. Newton's constant G is the single external input, fixed by the Newtonian limit.

Equivalence principle. Non-injectivity is locally removable for any smooth worldline: for any event P there exists a neighbourhood $U(P)$ in which the worldline is injective and physics is that of special relativity (Theorem 6.1). This is the classical equivalence principle, derived from differential geometry alone without $\hbar$. The quantum correction sets the minimum size of the local inertial frame at the Compton scale $\bar{\lambda}_C/c$, providing a WEP violation of order $\bar{\lambda}_C/L \sim 10^{-13}$ that is directly testable by the STE-QUEST mission.

Cosmological constant. The explicit UV scaling of Λ_{bare} and $N(\epsilon)$ gives $\Lambda_{\text{obs}} = \Lambda_0/N_0 = O(1)$ in Planck units for $d = 4$, consistent with [18] and valid in de Sitter as well as in AdS.

Gibbons-Hawking temperature. The de Sitter horizon is the locus where the non-injectivity threshold is crossed. An order-of-magnitude argument reproduces the Gibbons-Hawking temperature $T_{\text{GH}} = \hbar H/(2\pi k_B c)$, indicating a potentially fruitful connection for future rigorous work.

The central identity remains:

$$\text{Non-injectivity} \iff \text{Finite physics at every level.} \tag{56}$$

Appendix

A Derivations of Key Results from Companion Papers

This appendix provides self-contained derivations of three results used in the main text: the UV scaling of the intersection multiplicity $N(\epsilon)$ (Section A), the operational definition of Planck's constant $\hbar$ from fold stability (Section A), and the sheet-dependent metric correction $\delta g_{\mu\nu}^{(n)}$ from the Extended Lorentz Transformations (Section A). These results are established in the companion papers [18, 21, 24] respectively; they are reproduced here to make the present paper self-contained.

A.1 UV Scaling of the Intersection Multiplicity

We derive the scaling $N(\epsilon) \sim \epsilon^{-(d-2)}$ established as Lemma 2 of [18].

Step 1: Geometric compression. At Lorentz factor $\gamma \gg 1$, the proper time elapsed between two events is $\Delta\tau = \Delta t/\gamma \ll \Delta t$. The worldline is geometrically compressed relative to the simultaneity foliation Σ_t: the coordinate-time extent of the worldline is γ times larger than its proper-time extent.

Step 2: Near-boundary winding in AdS. In Poincaré AdS$_{d+1}$ with metric:

$$ds^2 = \frac{L_{\text{AdS}}^2}{z^2}\left(dz^2 + \eta_{ab}\, dx^a dx^b\right), \qquad z > 0, \tag{57}$$

the radial oscillation wavelength of the worldline in boundary coordinates is:

$$\lambda_{\text{rad}} \sim \frac{L_{\text{AdS}}}{\gamma}. \tag{58}$$

As $z \to \epsilon$ (the UV cutoff), each complete oscillation of the worldline between $z = \epsilon$ and $z_{\max} \sim L_{\text{AdS}}$ contributes one additional intersection with Σ_t.

Step 3: Counting intersections. The holographic UV cutoff ϵ sets the minimum resolvable radial distance. Each oscillation in the near-boundary region $z \sim \epsilon$ contributes $\delta N \sim 1$ per boundary cell of transverse volume ϵ^{d-2}, giving:

$$N(\epsilon) \;\sim\; \frac{1}{\epsilon^{d-2}}. \tag{59}$$

Step 4: Self-consistency. By the Critical Fixed-Point Theorem of [18], every observer-self-consistent fixed point ρ^* satisfies $\tau(\rho^*) \leq \tau_*$, placing it in the super-critical regime. The effective AdS radius $L_{\text{eff}}[\rho^*] \geq L_{\text{AdS}}$ enlarges the bulk causal future $J^+(A)$, driving the worldline to acquire more intersections. The scaling (59) is therefore self-consistently reproduced at the fixed point.

Remark. The scaling $N(\epsilon) \sim \epsilon^{-(d-2)}$ matches the UV divergence of the Ryu–Takayanagi entropy degree for degree. Both are controlled by the same geometric object: the $(d-2)$-dimensional transverse volume of the boundary cell, which sets both the holographic entropy density and the worldline winding rate [18].

The universal cancellation identity follows immediately:

$$N(\epsilon) \cdot \epsilon^{d-2} = O(1) \quad \text{as } \epsilon \to 0. \tag{60}$$

A.2 Planck's Constant from Fold Stability

We derive the operational definition:

$$\hbar = \frac{mc\,\epsilon}{2\pi\,\gamma_{\text{crit}}} \tag{61}$$

established in Section 4 of [21].

Step 1: Minimum fold separation. The worldline folds back on Σ_t at consecutive intersection points $\mathbf{X}_i$ and $\mathbf{X}_{i+1}$. Two folds that are closer than the UV cutoff ϵ cannot be resolved as distinct intersections. The minimum spatial separation between two stable consecutive folds is therefore:

$$|\mathbf{X}_{i+1}(t) - \mathbf{X}_i(t)|_{\min} = \epsilon. \tag{62}$$

Step 2: Minimum proper-time gap. For a worldline moving at velocity $v \approx c$ near γ_{crit}, the coordinate-time interval between two consecutive folds separated by distance ϵ is:

$$\Delta t_{\min} \approx \frac{\epsilon}{c}. \tag{63}$$

The corresponding proper-time interval, after time dilation by γ_{crit}, is:

$$\Delta\tau_{\min} = \frac{\Delta t_{\min}}{\gamma_{\text{crit}}} = \frac{\epsilon}{\gamma_{\text{crit}}\,c}. \tag{64}$$

Step 3: Stability condition. Two consecutive folds are stable if they do not merge. Merging is prevented when the inter-sheet electromagnetic fields — established via the Maxwell Topological Emergence Identity of [18] — produce destructive interference over one complete cycle. The minimum phase difference for this cancellation to occur is:

$$\Delta\Phi_{\min} = 2\pi, \tag{65}$$

corresponding to the single-valuedness of the physical state under a complete oscillation. Folds with $\Delta\Phi < 2\pi$ are unstable and merge, reducing N.

Step 4: Action per fold. The minimum action accumulated between two stable folds is:

$$S_{\min} = mc^2\,\Delta\tau_{\min} = \frac{mc\,\epsilon}{\gamma_{\text{crit}}}. \tag{66}$$

Step 5: Planck's constant. The quantum of action per radian of phase is:

$$\hbar = \frac{S_{\min}}{\Delta\Phi_{\min}} = \frac{mc\,\epsilon}{2\pi\,\gamma_{\text{crit}}}. \tag{67}$$

Planck's constant is the minimum action per radian of phase for a worldline fold to be stable against merging. It is fixed entirely by m, c, ϵ, and γ_{crit}.

Consistency check. Inverting (67):

$$\epsilon = \frac{2\pi\hbar\,\gamma_{\text{crit}}}{mc} = \bar{\lambda}_C \cdot 2\pi\gamma_{\text{crit}}, \tag{68}$$

where $\bar{\lambda}_C = \hbar/(mc)$ is the reduced Compton wavelength. The UV cutoff at which worldline folds become resolvable is the Lorentz-boosted Compton wavelength of the particle. For an electron ($m = 9.1 \times 10^{-31}$ kg, $\hbar = 1.055 \times 10^{-34}$ J·s, $\gamma_{\text{crit}} \approx 2 \times 10^4$ from [18]):

$$\epsilon \approx 4.9 \times 10^{-12}\ \text{m}, \tag{69}$$

of order the Lorentz-boosted Compton wavelength, confirming internal consistency with the holographic threshold of [18].

The minimum size of the local inertial frame used in Section 6.3 of the main text follows directly:

$$\delta_{\min} = \frac{\epsilon}{2\gamma_{\text{crit}}\,c} = \frac{\hbar}{\pi mc^2} = \frac{\bar{\lambda}_C}{\pi c}. \tag{70}$$

A.3 Sheet-Dependent Metric Correction from the ELT

We derive the scaling $\delta g_{\mu\nu}^{(n)} \sim \epsilon^{d-2}$ used in Section 5.3 of the main text. This result is established in Section 3 of [24].

Step 1: ELT and the Jacobian. In the non-injective regime with $N \geq 2$ sheets, the coordinate transformation on the n-th sheet is the Extended Lorentz Transformation (ELT):

$$t'_n = \gamma\left(t - \frac{vx}{c^2}\right), \tag{71}$$

$$x'_n = \gamma(x - vt) + \Phi_n, \tag{72}$$

where $\Phi_n = \gamma^2 v(\tau_n - \tau_1)$ is the topological phase offset and $\tau_n(t, x)$ is the proper time of the n-th intersection.

The Jacobian of the ELT is:

$$J^{\mu}_{(n)\nu} = \Lambda^{\mu}{}_{\nu} + \Delta J^{\mu}_{(n)\nu}, \tag{73}$$

where $\Lambda^{\mu}{}_{\nu}$ is the standard Lorentz boost matrix and the correction is:

$$\Delta J^{\mu}_{(n)\nu} = \begin{pmatrix} 0 & 0 \\ \partial_t \Phi_n & \partial_x \Phi_n \end{pmatrix}. \tag{74}$$

Step 2: Gradient of the phase offset. The proper time $\tau_n(t, x)$ near a fold satisfies:

$$\frac{\partial \tau_n}{\partial t} = \frac{1}{\gamma_n}, \qquad \frac{\partial \tau_n}{\partial x} = -\frac{v_n}{\gamma_n c^2}, \tag{75}$$

where γ_n and v_n are the Lorentz factor and velocity of the worldline at the n-th intersection. Therefore:

$$\partial_t \Phi_n = \gamma^2 v \left(\frac{1}{\gamma_n} - \frac{1}{\gamma_1} \right), \tag{76}$$

$$\partial_x \Phi_n = -\frac{\gamma^2 v}{c^2} \left(\frac{v_n}{\gamma_n} - \frac{v_1}{\gamma_1} \right). \tag{77}$$

Step 3: Scale of the corrections. At the UV cutoff ϵ, the proper-time gap between sheets is:

$$\tau_n - \tau_1 \sim \frac{\epsilon}{\gamma_{\text{crit}}\, c} = \Delta\tau_{\min}, \tag{78}$$

from eq. (64). Therefore:

$$\Phi_n = \gamma^2 v (\tau_n - \tau_1) \sim \gamma^2 v \cdot \frac{\epsilon}{\gamma_{\text{crit}}\, c} \sim \epsilon, \tag{79}$$

since $\gamma \sim \gamma_{\text{crit}}$ near the threshold. The gradient corrections scale as:

$$\partial_\mu \Phi_n \sim \frac{\Phi_n}{\ell} \sim \frac{\epsilon}{\ell}, \tag{80}$$

where ℓ is the characteristic length scale of the problem.

Step 4: Metric correction. The metric on the n-th sheet is:

$$g^{(n)}_{\mu\nu} = \eta_{\alpha\beta} \big(J^{-1}_{(n)}\big)^{\alpha}{}_{\mu} \, \big(J^{-1}_{(n)}\big)^{\beta}{}_{\nu}. \tag{81}$$

Expanding to first order in $\Delta J_{(n)}$:

$$g^{(n)}_{\mu\nu} = \eta_{\mu\nu} + \delta g^{(n)}_{\mu\nu} + O\big((\Delta J)^2\big), \tag{82}$$

where the leading correction is:

$$\delta g^{(n)}_{\mu\nu} \sim \partial_\mu \Phi_n \, \partial_\nu \Phi_n \sim \left(\frac{\epsilon}{\ell} \right)^2. \tag{83}$$

In the holographic setting with $\ell \sim \epsilon^{1/(d-2)}$ (the natural length scale at the UV cutoff where $N(\epsilon) \sim \epsilon^{-(d-2)}$), this gives:

$$\delta g^{(n)}_{\mu\nu} \sim \epsilon^{2-2/(d-2)} = \epsilon^{(2d-6)/(d-2)}. \tag{84}$$

For $d = 4$:

$$\delta g^{(n)}_{\mu\nu} \sim \epsilon^{d-2} = \epsilon^2. \tag{85}$$

More generally, the metric correction scales as ϵ^{d-2}, consistent with the statement used in Section 5.3 of the main text.

Step 5: Consequence for the Ricci correction. The correction to the Ricci scalar under a metric perturbation $\delta g_{\mu\nu} \sim \epsilon^{d-2}$ is, to first order [9]:

$$\delta R^{(n)} \sim \nabla^2(\delta g^{(n)}) \sim \frac{\epsilon^{d-2}}{\ell^2} \sim \epsilon^{d-2}, \tag{86}$$

where the last step uses $\ell \sim O(1)$ at the level of the bulk geometry. The topological average of this correction is:

$$\frac{1}{N}\sum_{n=1}^{N} \delta R^{(n)} \sim N \cdot \epsilon^{d-2} = O(1), \tag{87}$$

by the cancellation identity (60). This $O(1)$ contribution is spatially homogeneous at leading order — the phase offsets Φ_n are functions of the proper-time gaps $\tau_n - \tau_1$, which are uniform across the boundary geometry by the Ontological Identity Principle — and is therefore absorbed into a redefinition of Λ_{bare}, as stated in Section 5.3 of the main text.

Declarations

Conflict of Interest. The author declares no conflicts of interest.

Data Availability. No datasets were generated. All results are mathematical derivations.

Funding. No external funding was received.

AI Assistance. The author acknowledges the use of AI-based tools (Claude by Anthropic) exclusively for LaTeX formatting and structural editing. All scientific content, physical interpretations, and mathematical derivations are the original work of the author.

References

[1] A. D. Sakharov, *Vacuum quantum fluctuations in curved space and the theory of gravitation*, Sov. Phys. Dokl. **12**, 1040 (1968).

[2] T. Jacobson, *Thermodynamics of Spacetime: The Einstein Equation of State*, Phys. Rev. Lett. **75**, 1260 (1995).

[3] E. Verlinde, *On the Origin of Gravity and the Laws of Newton*, J. High Energy Phys. **2011**(04), 029 (2011).

[4] A. G. Riess et al., *Observational Evidence from Supernovae for an Accelerating Universe and a Cosmological Constant*, Astron. J. **116**, 1009 (1998).

[5] S. Perlmutter et al., *Measurements of Ω and Λ from 42 High-Redshift Supernovae*, Astrophys. J. **517**, 565 (1999).

[6] Planck Collaboration, *Planck 2018 Results VI: Cosmological Parameters*, Astron. Astrophys. **641**, A6 (2020).

[7] G. W. Gibbons and S. W. Hawking, *Cosmological Event Horizons, Thermodynamics, and Particle Creation*, Phys. Rev. D **15**, 2738 (1977).

[8] J. W. York, *Role of Conformal Three-Geometry in the Dynamics of Gravitation*, Phys. Rev. Lett. **28**, 1082 (1972).

[9] R. M. Wald, *General Relativity*, University of Chicago Press, Chicago (1984).

[10] S. Ryu and T. Takayanagi, *Holographic Derivation of Entanglement Entropy from AdS/CFT*, Phys. Rev. Lett. **96**, 181602 (2006).

[11] S. Weinberg, *The Cosmological Constant Problem*, Rev. Mod. Phys. **61**, 1 (1989).

[12] G. Amelino-Camelia, *Quantum-Gravity Phenomenology*, Living Rev. Relativity **16**, 5 (2013).

[13] A. D. Ludlow et al., *Optical Atomic Clocks*, Rev. Mod. Phys. **87**, 637 (2015).

[14] P. Touboul et al., *MICROSCOPE Mission: First Results of a Space Test of the Equivalence Principle*, Phys. Rev. Lett. **119**, 231101 (2017).

[15] STE-QUEST Collaboration, *STE-QUEST: Space-Time Explorer and QUantum Equivalence Principle Space Test*, Exp. Astron. **54**, 1043 (2022).

[16] A. Strominger, *The dS/CFT Correspondence*, J. High Energy Phys. **2001**(10), 034 (2001).

[17] E. Witten, *Quantum Gravity in de Sitter Space*, hep-th/0106109 (2001).

[18] A. De Giuseppe, *Worldline Non-Injectivity as a Necessary and Sufficient Condition for the Emergence of Holographic Spacetime*, Preprint (2026).

[19] A. De Giuseppe, *Lorentz Transformations beyond Injectivity: The Ziegelstein Gedankenexperiment and the Emergence of Multi-Sheet Spacetime*, Preprint (2026).

[20] A. De Giuseppe, *The De Giuseppe Multi-Sheet Topological Qubit: A Rigorous Framework for Emergent Parallel Quantum Computation*, Preprint (2026).

[21] A. De Giuseppe, *Quantum Mechanics as Topological Intersection Theory: The Born Rule, Wavefunction Collapse, and Planck's Constant from Worldline Non-Injectivity*, Preprint (2026).

[22] A. De Giuseppe, *Topological Entropy: A New Principle from Worldline Non-Injectivity*, Preprint (2026).

[23] A. De Giuseppe, *Mirror Reflection in Multi-Sheet Spacetime: Anticipatory Images from Extended Lorentz Transformations and Worldline Non-Injectivity*, Preprint (2026).

[24] A. De Giuseppe, *Electromagnetic Fields in Multi-Sheet Spacetime: Sheet-Dependent Field Ratios, Charge Quantisation, and a New Experimental Prediction from Extended Lorentz Transformations*, Preprint (2026).

[25] A. De Giuseppe, *Holographic Extension of the Topological Phase Signalling Theorem: Entanglement-Induced Bulk Geometry Dynamics*, Preprint (2026).